5.1

Go for the Curve!

Comparing Linear and Exponential Functions

LEARNING GOALS

In this lesson, you will:

- Construct and identify linear and exponential functions from sequences.
- Compare graphs, tables, and equations of linear and exponential functions.
- Construct a linear function from an arithmetic sequence.
- Construct an exponential function from a geometric sequence.
- Compare formulas for simple interest and compound interest.

KEY TERMS

- simple interest
- compound interest

It might seem unfair, but many banks will charge you money for not having money. And they'll pay you money if you have a lot of it. In 2008, U.S. banks made about $24 billion in overdraft fees—a 35% increase from 2006. By contrast, personal interest income in 2008 was over $1 trillion!

When you deposit money in a bank account which accrues interest, your money doesn't just sit there, waiting for you to withdraw it. The bank lends this money to people who want to buy cars, houses, and pay for college.

Banks collect interest on these loans and reward you for your contribution. The more money you have in an interest-earning account, the more you are rewarded!

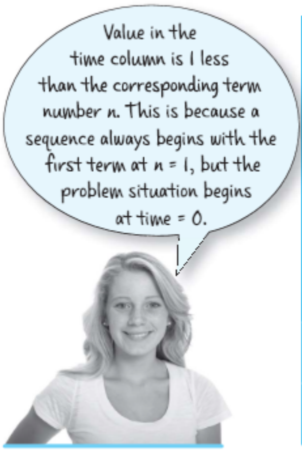
PROBLEM 1 Let's Build a Formula for Simple Interest

Nico considers depositing money into an account that earns *simple interest* each year.

In a **simple interest** account, the interest earned at the end of each year is a percent of the original deposited amount (also known as the original principal). If Nico deposits $500 into an account that earns 2% simple interest each year, 2% of the original principal, or $500(0.02), is added to the account each year.

The balance after 1 year would be $500 + $500(0.02), or $510. The balance after 2 years would be $510 + $500(0.02), or $520, and so on.

1. Suppose that Nico deposits $1000 into an account that earns 5% simple interest each year. Complete the table to show Nico's account balance after each year. Show your work. The first three rows have been completed for you.



Term Number (n)	Time (years)	Interest Earned (dollars)	Account Balance (dollars)
1	0	0	1000
2	1	1000(0.05) = 50	1000 + 1000(0.05) = 1050
3	2	1000(0.05) = 50	1050 + 1000(0.05) = 1100
4			
5			
6			

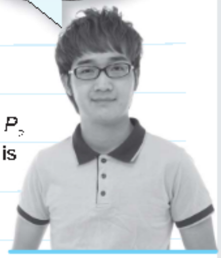
2. Is this problem situation a sequence? If so, describe the sequence type and how you know. If it is not a sequence, explain why not.

3. Suppose P_t represents the account balance after t years. So, P_0 represents the account balance after 0 years (the original principal), P_1 represents the account balance after 1 year, P_2 represents the account balance after 2 years, and so on. If r represents the interest rate, what does $P_0 \cdot r$ represent?

The amount in the account after 1 year, P_1, is equal to the initial amount plus the interest earned.

Amount after 1 year = Initial amount + Interest earned

$$P_1 = P_0 + P_0 \cdot r$$



You can use the formula for P_1 to determine the formula for P_2 in terms of P_0. The amount in the account after 2 years, P_2, is equal to the amount in the account after 1 year plus the interest earned.

Amount after 2 years = Amount after 1 year + Interest earned

$$P_2 = P_1 + P_0 \cdot r$$

$$P_2 = [P_0 + P_0 \cdot r] + P_0 \cdot r$$

You can continue this sequence to determine the balance in the account after 3 years, 4 years, and so on, in terms of P_0.

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4. Write a formula in terms of P_0 to represent the balance in the account after:

a. 3 years.

$P_3 = P_2 + P_0 \cdot r$

$P_3 =$ ______________________________ + ________

b. 4 years.

$P_4 = P_3 + P_0 \cdot r$

$P_4 =$ ______________________________ + ________

c. 5 years.

$P_5 =$ ______________________________ + ________

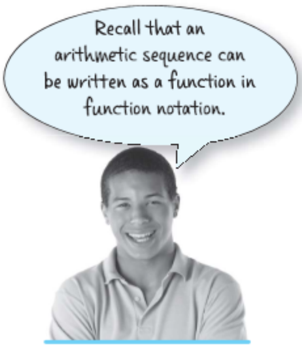
5. Is this an arithmetic or geometric sequence? How do you know? Determine the common difference or common ratio from the formulas in Question 4.

6. Substitute the original principal P_0, and the common difference or common ratio into the explicit formula for the sequence.

7. Remember that t represents the time in years. Since t begins at 0 and the term number n begins at 1, t is always 1 less than n. How can you represent t algebraically in terms of n?

$t =$ ______________

8. Let $P(t)$ represent the account balance after t years. Use your answers in Questions 6 and 7 to write the formula for the balance $P(t)$ in the simple interest account as a function of time t.



9. Write Nico's balance in the simple interest account described in Question 1 as a function of time t. Recall that the original principal is $1000 and the interest rate is 5%. Use the function to verify the values in your table.

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10. Determine which function family represents the simple interest formula. Explain your reasoning.

11. Use the function you wrote in Question 9 to determine Nico's account balance after:

   a. 8 years.

   b. 100 years.

12. Use the **intersection** feature of a graphing calculator to determine the number of years it will take for the balance in Nico's account to:

   a. reach $1600.

   b. double.

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PROBLEM 2 Now Let's Build a Formula for Compound Interest

Raul considers depositing money into an account that earns *compound interest* each year.

In a **compound interest** account, the interest earned at the end of each year is a percent of the account balance at the beginning of the year. For example, if \$500 is deposited into an account that earns 2% compound interest each year, the balance after 1 year would be \$500 + \$500(0.02) = \$510, the balance after 2 years would be \$510 + \$510(0.02) = \$520.20, and so on.

1. Suppose that Raul deposits \$1000 into an account that earns 5% compound interest each year. Complete the table to show Raul's account balance after each year. Show your work. The first three rows have been completed for you.

Term Number (n)	Time (years)	Interest Earned (dollars)	Account Balance (dollars)
1	0	0	1000
2	1	1000(0.05) = \$50	1000 + 1000(0.05) = 1050
3	2	1050(0.05) = 52.5	1050 + 1050(0.05) = 1102.5
4			
5			
6			

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Suppose that P_0 represents Raul's original principal, and r represents the interest rate in the compound interest account. So, the formula

$$P_1 = P_0 + P_0 \cdot r$$

represents the balance in the account in terms of P_0 after 1 year. You can use the Distributive Property to rewrite this formula as

$$P_1 = P_0(1 + r)$$

2. Use the Distributive Property to write a formula to represent the balance in the account in terms of P_0 after the given number of years. The first one has been done for you.

a. 2 years

$P_2 = P_1 + P_1 \cdot r$

$P_2 = P_1(1 + r)$

$P_2 = [P_0(1 + r)](1 + r) = P_0(1 + r)(1 + r)$

b. 3 years

$P_3 = P_2 + P_2 \cdot r$

$P_3 =$

$P_3 =$

c. 4 years

$P_4 = P_3 + P_3 \cdot r$

$P_4 =$

$P_4 =$

3. Is this an arithmetic or geometric sequence? How do you know? Determine the common difference or common ratio from the formulas in Question 2.

4. Substitute Raul's original principal, P_0, and the common difference or common ratio into the explicit formula for the sequence.

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5. Let $P(t)$ represent Raul's account balance after t years. Remember that time t is 1 less than the term number n. Write the formula for the balance $P(t)$ in the compound interest account as a function of time t.

6. Write the balance in Raul's compound interest account as a function of time t. Recall that the original principal is \$1000 and the interest rate is 5%. Use the function to verify the values in your table.

7. Determine which function family represents the compound interest formula. Explain your reasoning.

8. Use the function you wrote in Question 6 to determine the account balance after:

a. 8 years.

b. 100 years.

9. Use the **intersection** feature of a graphing calculator to determine the number of years it will take for the balance in Raul's account to:

a. reach $1600.

b. double.

Talk the Talk

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You now know the formulas for simple and compound interest.

The simple interest formula is

$$P_t = P_0 + (P_0 \cdot r)t$$
$$= P_0(1 + rt)$$

where P_t represents the balance in the account after t years, P_0 represents the original principal, and r represents the interest rate each year (written as a decimal).

The compound interest formula is

$$P_t = P_0 \cdot (1 + r)^t$$

where P_t represents the balance in the account after t years, P_0 represents the original principal, and r represents the interest rate each year (written as a decimal).

1. Use the simple and compound interest formulas from the situations for Nico's simple interest account and Raul's compound interest account to complete the table. Round the values to the nearest cent.

Quantity	Time	Simple Interest Balance	Compound Interest Balance
Units			
Expression			
	0		
	1		
	2		
	8		
	100		

2. Terrell is looking for some financial advice. He has the option to deposit $1000 into the simple interest account just like Nico's account, or a compound interest account just like Raul's account. The compound interest account would cost him a one-time start-up fee of $200. The simple interest account is free. Into which account would you tell Terrell to put his money and why?

3. Graph the simple interest and compound interest functions on your calculator. Then, sketch the graphs on the given grid. Use the bounds [0, 40] × [0, 6000].

4. What is the average rate of change for the simple interest function? Explain how you know.

5. Determine the average rate of change between each pair of values given for the compound interest function.

- Between $t = 0$ and $t = 1$:
- Between $t = 1$ and $t = 2$:
- Between $t = 2$ and $t = 8$:
- Between $t = 8$ and $t = 100$:

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6. Compare the average rates of change for the simple and compound interest accounts.

   a. What do you notice?

   b. What does this tell you about the graphs of linear and exponential functions?

Be prepared to share your solutions and methods.